References

[1] D. N. Zuckerman and P. Diament, "Rank reduction of ill-conditioned matrices in waveguide junction problems," to be published.

[2] T. A. Abele *et al.*, "A high-capacity digital communication system using TE_{01} transmission in circular waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 326–333, Apr. 1975.

[3] E. A. Marcatili and D. L. Bisbee, "Band-splitting filter," *Bell Syst. Tech. J.*, vol. 40, pp. 197–212, 1961.

[4] S. Iiguchi, "Michelson interferometer type hybrid for circular TE_{01} wave and its application to band-splitting filter," *Rev. Elec. Comm. Lab.*, vol. 10, pp. 631–642, 1962.

[5] U. Unrau, "Band-splitting filters in oversized rectangular wave-guide," *Electron. Lett.*, vol. 9, pp. 30–31, Jan. 25, 1973.

[6] J. J. Taub and J. Cohen, "Quasi-optical waveguide filters for millimeter and submillimeter wavelengths," *Proc. IEEE*, vol. 54, pp. 647–656, 1966.

[7] B. Wardrop, "A quasi-optical directional coupler," *Marconi Rev.*, second quarter, pp. 159–169, 1972.

[8] U. Unrau, "Exact analysis of directional couplers and dielectric coated mirrors in overmoded waveguide," presented at the European Microwave Conf., Brussels, Belgium, 1973 (paper B 4.3).

[9] S. A. Schelkunoff, "Some equivalence theorems of electromagnetics and their application to radiation problems," *Bell Syst. Tech. J.*, vol. 15, pp. 92–112, 1936.

[10] A. Ben-Israel and T. N. E. Greville, *Generalized Inverses: Theory and Applications.* New York: Wiley, 1974.

[11] C. J. Bouwkamp, "Scattering characteristics of a cross-junction of oversized waveguides," *Philips Tech. Rev.*, vol. 32, pp. 165–178, 1971.

[12] M. Becker, *The Principles and Applications of Variational Methods.* Cambridge, MA: M.I.T. Press, 1964.

Propagation Losses of Guided Modes in an Optical Graded-Index Slab Waveguide with Metal Cladding

MASAMITSU MASUDA, AKIHITO TANJI, YASUHIRO ANDO, AND JIRO KOYAMA

***Abstract*—Analytical results for propagation losses of guided modes in a graded-index slab waveguide (GISW) with metal cladding are presented. When the permittivity in the guiding layer decreases linearly away from the metal surface, the attenuation constant α_G of well-guided modes, TE and TM, is approximately proportional to only the ratio $(\Delta\varepsilon_i/\varepsilon_0)/d_i$, where $\Delta\varepsilon_i$ is the increment in the permittivity at the metal surface in the direction of the polarization of optical waves, d_i is the diffusion depth in this direction, and ε_0 is the permittivity of free space.**

I. Introduction

ELECTROOPTIC crystal, such as $LiNbO_3$ or $LiTaO_3$, is very promising for use as the substrate of an integrated optical circuit. Recently, several experiments have been reported on the techniques for fabricating optical guides in these crystals, which consist of diffusing suitable metal ions into the substrate [1], [2] and out-diffusing Li_2O from the surface [3]. These methods yield a graded-index slab waveguide (GISW) instead of a step-index slab waveguide (SISW) in which the guiding layer has the space-invariant permittivity. Analyses of guided modes in the GISW have been made for the exponential [4] and linear [5] profiles of the permittivity. To interact the guided modes with low-frequency electromagnetic fields, metal electrodes with planar structures are generally needed. The GISW should become lossy due to the metal cladding on the surface. The effects of metal cladding have been examined for the SISW in homogeneous media in order to form a mode filter and an optical strip line [6]–[8]. On the other hand, the guiding properties of a metal-clad GISW, especially propagation losses of guided modes, have not been elucidated.

In this paper, propagation losses of guided modes in the GISW with metal cladding are analyzed under the assumption that the permittivity in the guiding layer decreases linearly away from the metal surface, taking the anisotropy of the electrooptic crystal into account. The effective thickness of the guiding layer is derived approximately. The attenuation constant α_G of guided modes in a metal-clad GISW is estimated in comparison with the attenuation constant α_S of guided modes in a metal-clad SISW.

Numerical solutions of the dispersion equation are given for both TE and TM modes in c-plate $LiNbO_3$.

Manuscript received February 23, 1976; revised January 14, 1977.

M. Masuda, A. Tanji, and J. Koyama are with the Department of Electronics, Faculty of Engineering, Osaka University, Yamada-Kami, Suita 565, Osaka, Japan.

Y. Ando is with Musashino Electrical Communication Laboratory, Nippon Telegraph and Telephone Public Corporation, Tokyo, Japan.

II. Analysis

A. Derivation of the Dispersion Equation

In our analysis, we assume that the optical wave varies as $\exp j(\omega t - k_z z)$ and that all quantities are independent of x,

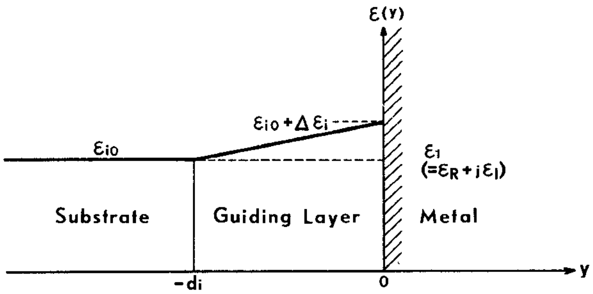

Fig. 1. The profiles of the permittivities along the depth in a metal-clad GISW. The guiding layer and the substrate below $y = 0$ consist of electrooptic crystal. The subscript i denotes x, y, or z, and d_i is the diffusion depth.

i.e., $\partial/\partial x = 0$. The profile of the permittivities is shown in Fig. 1 and is given for $i = x,y,z$, as follows:

$$\begin{aligned} \varepsilon_1 &= \varepsilon_R + j\varepsilon_I, && y \geq 0 \\ \varepsilon_i(y) &= \varepsilon_{i0} + \Delta\varepsilon_i(1 + y/d_i), && -d_i \leq y \leq 0 \\ \varepsilon_i(y) &= \varepsilon_{i0}, && y \leq -d_i \end{aligned} \tag{1}$$

where d_i is the diffusion depth. Since the frequency of the propagating light wave is lower than the plasma frequency of the metal, ε_R and ε_I have negative values and the dielectric slab with the metal cladding becomes lossy. When c-plate $LiNbO_3$ is, for example, in-diffused with metal ions, then we have $d_y \neq d_z$, $\Delta\varepsilon_y \neq \Delta\varepsilon_z$, and $\varepsilon_{y0} \neq \varepsilon_{z0}$. The resulting expressions are so lengthy that we deal with the case of $d_y = d_z$ for simplicity in this section.

Assuming that $\varepsilon_z(y)$ changes slowly compared with the optical wavelength λ, $(d/dy)[\ln \varepsilon_z(y)]$ may be neglected for TM modes. The following equations can be obtained in the guiding layer $-d_i \leq y \leq 0$ if only the terms of the first order in $\Delta\varepsilon_y$ and $\Delta\varepsilon_z$ are retained:

$$\frac{\partial^2 G_x}{\partial y^2} + a^2(y + y_0)G_x = 0 \tag{2}$$

where

$$G_x = \begin{cases} E_x, & \text{for TE modes} \\ H_x/\sqrt{\varepsilon_z(y)/\varepsilon_0}, & \text{for TM modes} \end{cases} \tag{3}$$

$$a^2 = \left(\frac{\Delta\varepsilon_i/\varepsilon_0}{d_i}\right) k_s^2 \left[\frac{1}{1 - \Delta K_y + \Delta K_z} \cdot \left\{K + (\Delta K_z - \Delta K_y)\frac{1}{\Delta\varepsilon_y/\varepsilon_0}\left(\frac{k_y}{k_s}\right)^2\right\}\right]$$

$$y_0 = \left(\frac{k_y}{a}\right)^2$$

$$k_y^2 = (\varepsilon_{i0} + \Delta\varepsilon_i)k_s^2/\varepsilon_0 - K(1 - \Delta K_y + \Delta K_z)k_z^2 \tag{4}$$

and

$$\begin{aligned} &K = 1, \quad \Delta K_y = \Delta K_z, \quad i = x, \quad \text{for TE modes} \\ &K = \varepsilon_{z0}/\varepsilon_{y0}, \quad \Delta K_y = \Delta\varepsilon_y/\varepsilon_{y0}, \\ &\Delta K_z = \Delta\varepsilon_z/\varepsilon_{z0}, \quad i = y, \quad \text{for TM modes.} \end{aligned} \tag{5}$$

k_s equals $2\pi/\lambda$ and k_y is the transverse wavenumber of the guided modes on the metal surface at $y = 0$.

Considering that Re (y_0) ranges between d_i and zero for guided modes, the exact solutions of (2) can be expressed in terms of Airy functions [9]. For Re $(y + y_0) > 0$, we obtain

$$G_x = C_2 Ai(-\zeta_1) + C_3 Bi(-\zeta_1) \tag{6}$$

where C_2 and C_3 are arbitrary coefficients and $\zeta_1 = a^{2/3}(y + y_0)$. On the other hand, G_x should be a monotonically decaying function over the range of Re $(y + y_0) < 0$ within the guiding layer; therefore, the arguments of Airy functions must be replaced by ζ_2, where $\zeta_2 = -a^{2/3}(y + y_0)$.

In the substrate below $y = -d_i$ and in the metal above $y = 0$, all fields of guided modes decay exponentially as follows:

$$G_x = C_4 \exp(\gamma_3 y), \qquad y \leq -d_i \tag{7}$$

$$G_x = C_1 \exp(-\gamma_1 y), \qquad y \geq 0 \tag{8}$$

$$\gamma_3^2 = K(k_z^2 - \varepsilon_{i0}k_s^2/\varepsilon_0) \tag{9}$$

$$\gamma_1^2 = k_z^2 - \varepsilon_1 k_s^2/\varepsilon_0. \tag{10}$$

Introducing the boundary condition that tangential components are continuous on two interfaces, at $y = 0$ and $y = -d_i$, to eliminate arbitrary coefficients C_1, C_2, C_3, and C_4, we can obtain the dispersion equation if the representations of Airy functions in terms of Bessel functions are used, as follows:

$$\frac{J_{2/3}(u_0) - g_1 J_{-1/3}(u_0)}{J_{-2/3}(u_0) + g_1 J_{1/3}(u_0)} = \frac{I_{2/3}(u_d) + g_2 I_{-1/3}(u_d)}{I_{-2/3}(u_d) + g_2 I_{1/3}(u_d)} \tag{11}$$

where for TE modes

$$g_1 = \gamma_1/k_y \qquad g_2 = 1$$

for TM modes

$$\begin{aligned} g_1 &= \left\{\frac{\varepsilon_{z0}}{\varepsilon_1}(1 + \Delta K_z)\right\}\frac{\gamma_1}{k_y} + \frac{1}{2}\left(\frac{\Delta K_z}{1 + \Delta K_z}\right)\frac{1}{k_y d_y} \\ g_2 &= 1 - \frac{1}{2}\Delta K_z \frac{1}{\gamma_3 d_y} \end{aligned} \tag{12}$$

and

$$u_0 = \tfrac{2}{3}a y_0^{3/2} \qquad u_d = \tfrac{2}{3}a(d_i - y_0)^{3/2}.$$

B. The Effective Thickness of the Guiding Layer

The exact solutions of (11) can be obtained by numerical computation, as shown in the next section. We are interested in only the dominant factors, though, which influence the attenuation constant. To estimate the effective thickness of the guiding layer, the metal is assumed to be a perfect conductor; accordingly the GISW becomes lossless. The validity of this assumption can be seen by the following argument: the difference between ε_R and ε_{i0} is much larger than $\Delta\varepsilon_i$, so that there is little penetration of the field into the metal.

The right-hand side of (11) is a monotonically increasing function for the variable u_d. Its value equals zero at the cutoff of the guided modes, where k_z reduces to $k_s\sqrt{\varepsilon_{i0}/\varepsilon_0}$ $(i = x,y)$. When optical waves are poorly guided, its value is small. Its value increases rapidly and then is very close to unity for well-guided modes. Using asymptotic expressions of Bessel functions, the approximate expression of (11) can be reduced to the following relation for well-guided modes:

$$u_0 \simeq (q - \tfrac{1}{4})\pi \qquad (13)$$

where the mode number q corresponds to the number of extrema which each field component has within the guiding layer. If y_t is defined by y_0 under the assumption that the metal is a perfect conductor, we obtain

$$y_t \simeq \left\{ \tfrac{3}{2}(q - \tfrac{1}{4})\pi K^{-1/2} \frac{1}{k_s} \left(\frac{d_i}{\Delta\varepsilon_i/\varepsilon_0} \right)^{1/2} \right\}^{2/3}. \qquad (14)$$

It is easily understood from (2) that the surface of $y = -y_t$ is the turning surface in the lossless GISW. The field function $G_x(y)$ becomes oscillatory over the range $-y_t \le y \le 0$, and then reduces to an exponentially decaying function below $y = -y_t$. It is possible to regard y_t as the effective thickness of the guiding layer in the lossless GISW. Even if the metal has the complex permittivity in the actual metal-clad GISW, the effective thickness of the guiding layer should be close to that for the above lossless GISW.

C. Estimation of the Attenuation Constant

We will estimate the attenuation constant of guided modes, in comparison with that for the metal-clad SISW. Two-dimensional analysis of the SISW [7], [8] indicates that the attenuation constant α_S is proportional to $(q\lambda)^2/b^3$ where b is the thickness of the guiding layer which has the space-invariant permittivity $\varepsilon_{i0} + \Delta\varepsilon_i$. α_S is scarcely influenced by the increment of the permittivity $\Delta\varepsilon_i$ under the condition $\Delta\varepsilon_i \ll \varepsilon_{i0}$.

The effective thickness y_t of the GISW in (14) corresponds to b. It is implied in (13) or (14) that $(q - \frac{1}{4})$ in the GISW should correspond to q in the SISW; in other words, a linear distribution of the permittivity in the GISW brings the constant phase shift $\pi/2$ to guided modes. Consequently, comparing the GISW with the SISW, we have the following corresponding relations:

$$y_t \leftrightarrow b \quad \text{and} \quad (q - \tfrac{1}{4}) \leftrightarrow q. \qquad (15)$$

Replacing b and q in the proportional expression $\alpha_S \propto (q\lambda)^2/b^3$ by y_t and $(q - \frac{1}{4})$, respectively, the attenuation constant α_G for the metal-clad GISW can be estimated:

$$\alpha_G \propto \frac{\Delta\varepsilon_i/\varepsilon_0}{d_i}, \qquad i = x,y. \qquad (16)$$

Since the effective thickness varies with $\{(q - \frac{1}{4})\lambda\}^{2/3}$, as indicated in (14), α_G is independent of the mode number q in the resulting expression (16). This is an essential feature of the GISW.

III. Numerical Solutions and Discussions

The dispersion equation (11) was numerically solved by the conventional Newton–Raphson method. Calculations were also made for aluminum and c-plate $LiNbO_3$ with the c axis along y. We assumed $\varepsilon_{x0}/\varepsilon_0 = \varepsilon_{z0}/\varepsilon_0 = 2.29^2$, $\varepsilon_{y0}/\varepsilon_0 = 2.20^2$, and $\varepsilon_1/\varepsilon_0 = -27.0 - j17.6$ for the wavelength 0.6328 μm of the He–Ne laser light [10]. Fig. 2 shows the dependency of α_G on $\Delta\varepsilon_i/\varepsilon_0$ where the parameter is the diffusion depth d_i and where we assumed $\Delta\varepsilon_z/\varepsilon_0 = 1.05 \times 10^{-2}$. Except in the neighborhood of the cutoff, the attenuation of well-guided modes increases linearly with $\Delta\varepsilon_i/\varepsilon_0$ for any diffusion depth. α_G is not dependent on the mode number q,

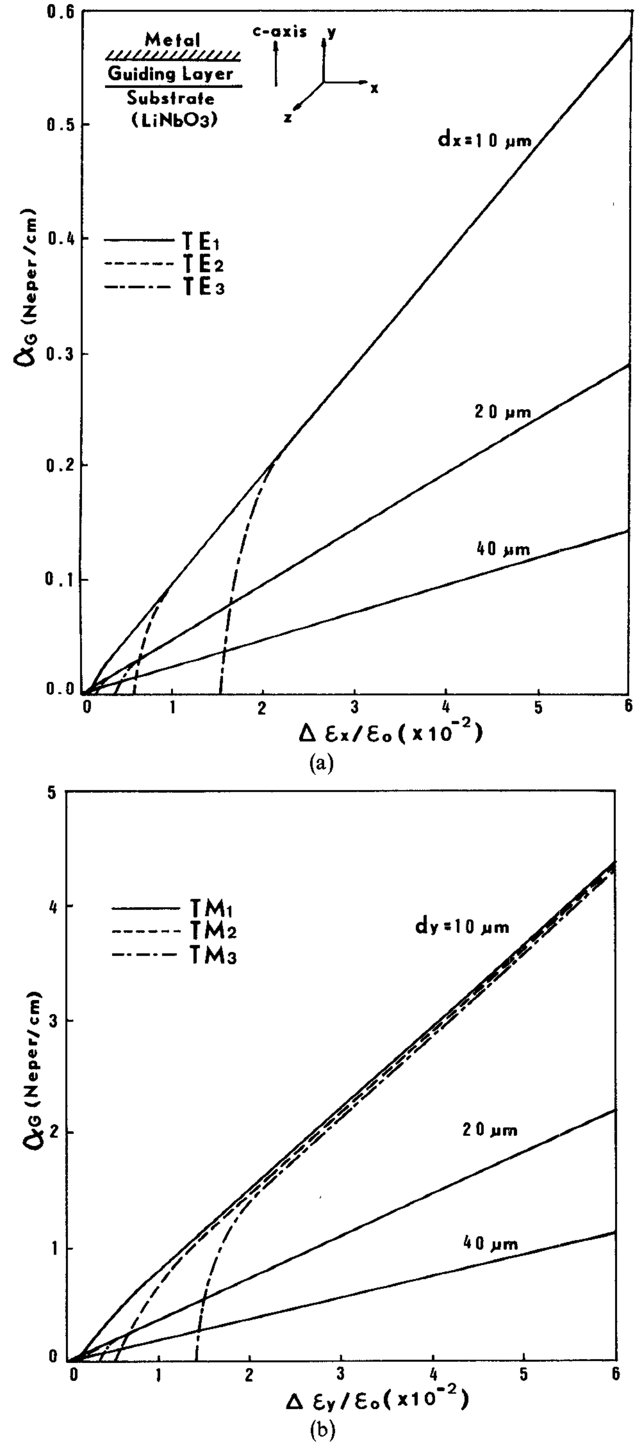

Fig. 2. Attenuation constants of guided modes in a metal-clad GISW. (a) TE modes. (b) TM modes. Calculations were made for the GISW in c-plate $LiNbO_3$ with aluminum surface cladding. The given values were $\varepsilon_{x0}/\varepsilon_0 = \varepsilon_{z0}/\varepsilon_0 = 2.29^2$, $\varepsilon_{y0}/\varepsilon_0 = 2.20^2$, and $\varepsilon_1/\varepsilon_0 = -27.0 - j17.6$ for the wavelength 0.6328 μm. Also we assumed $\Delta\varepsilon_z/\varepsilon_0 = 1.05 \times 10^{-2}$ and $d_y = d_z$.

but decreases for the higher order modes by a small amount. This tendency is particularly clear for TM modes in a thinner guide of $d_y = 10\ \mu$m.

The attenuation decreases rapidly near the cutoff, as shown in Fig. 2. This is caused by the fact that the concentration of fields on the metal surface is lowered, as guided modes are close to the cutoff.

The attenuation rate caused by the metal cladding is larger for TM modes than TE modes. If $\Delta\varepsilon_x = \Delta\varepsilon_y$ holds for well-guided modes, the attenuation of the TM modes is about eight times as much as that of the TE modes, except the lowest TM mode which behaves like a surface plasma wave [7]. The anisotropy of electrooptic crystal is not a dominant factor influencing the attenuation, but other settings of the c axis change the value of α_G slightly. When the c axis lies, for example, in the x direction, α_G for TE modes decreases by 2.4 percent, while for TM modes it increases by 10.5 percent. The above discussions have been made for the case of $d_y = d_z$. However, d_y and d_z are, in general, different from each other. In the most extreme case, the refractive index in the z direction is unchanged, that is, we have $d_z = 0$ and $\Delta\varepsilon_z = 0$, simultaneously. This situation can be realized by out-diffusing c-plate $LiNbO_3$ [3]. α_G for TM modes in this case is smaller than in the case of Fig. 2 by a few percent. Our results, which were obtained under the assumption that $d_y = d_z$, are scarcely influenced by $\Delta\varepsilon_z$ and the difference of the diffusion depth in different directions.

IV. Conclusion

We have discussed analytically the results of propagation losses of guided modes in a metal-clad GISW. In conclusion, the attenuation constant α_G for well-guided TE and TM modes in this optical waveguide is approximately proportional to only the ratio $(\Delta\varepsilon_i/\varepsilon_0)/d_i$. This feature of the metal-clad GISW, which is remarkably different from the feature for a metal-clad SISW, is caused by the variation of the effective thickness of the guiding layer with respect to q, λ, $\Delta\varepsilon_i$, and d_i.

Even if the profile of the permittivity in a diffused waveguide has the form of a complimentary error, a Gaussian or an exponential function, as observed in the actual guiding layer, it should be possible to find a straight line fitting each function to the first-order approximation. Therefore, the results presented in this paper may hold for an actual profile of the permittivity in the GISW.

Acknowledgment

The authors are grateful to Dr. H. Nishihara for his critical reading of the manuscript.

References

[1] R. V. Schmidt and I. P. Kaminow, "Metal-diffused optical waveguides in $LiNbO_3$," *Appl. Phys. Lett.*, vol. 25, pp. 458–460, Oct. 1974.
[2] J. Noda, T. Saku, and N. Uchida, "Fabrication of optical waveguiding layer in $LiTaO_3$ by Cu diffusion," *Appl. Phys. Lett.*, vol. 25, pp. 308–310, Sept. 1974.
[3] I. P. Kaminow and J. R. Carruthers, "Optical waveguiding layers in $LiNbO_3$ and $LiTaO_3$," *Appl. Phys. Lett.*, vol. 25, pp. 326–328, Apr. 1973.
[4] E. M. Conwell, "Modes in optical waveguides formed by diffusion," *Appl. Phys. Lett.*, vol. 23, pp. 328–329, Sept. 1973; also "Modes in anisotropic optical waveguides formed by diffusion," *IEEE J. Quantum Electron.*, vol. QE-10, pp. 608–612, Aug. 1974.
[5] D. Marcuse, "TE modes of graded-index slab waveguides," *IEEE J. Quantum Electron.*, vol. QE-9, pp. 1000–1006, Oct. 1973.
[6] Y. Takano and J. Hamasaki, "Propagating of a metal-clad-dielectric-slab waveguide for integrated optics," *IEEE J. Quantum Electron.*, vol. QE-8, pp. 206–212, Feb. 1972.
[7] I. P. Kaminow, W. L. Mammel, and H. P. Waber, "Metal-clad optical waveguides: Analytical and experimental study," *Appl. Opt.*, vol. 13, pp. 396–405, Feb. 1974.
[8] Y. Yamamoto, T. Kamiya, and H. Yanai, "Characteristics of optical guided modes in multilayer metal-clad planar optical guide with low-index dielectric bufferlayer," *IEEE J. Quantum Electron.*, vol. QE-11, pp. 729–736, Sept. 1975.
[9] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables.* New York: Dover, 1965, sec. 10.4.
[10] S. Iida, K. Ohno, I. Kanmae, H. Kumagai, and S. Sawada, *Tables of Physical Constants.* Tokyo, Japan: Asakura Book Co., 1969, sec. 5.5.